

CSMC CIMC Prep 9:

1. A triangle with side lengths of 3, 4, and 5 has the following five properties
 - a. The side lengths are integers
 - b. The lengths of its two shortest sides differ by one
 - c. The length of the longest side and the semi-perimeter differ by one
 - d. The area is an integer
 - e. The perimeter is less than 200

Determine all triangles that have these five properties

2. A total of 2015 tickets, numbered 1, 2, 3, 4,, 2014, 2015 are placed in an empty bag. Andrew removes ticket "a" from the bag. Betty then removes ticket "b" from the bag. Finally Chris removes ticket "c" from the bag. Given that $a < b < c$ and $a + b + c = 2018$, how many ways can this happen?

3. In a psychology experiment, an image of a cat or an image of a dog is flashed briefly onto a screen and then Andrew is asked to guess whether the image showed a cat or a dog. This process is repeated a large number of times with an equal number of images of cats and images of dogs shown. If Andrew is correct 95% of the time when he guesses "dog" and 90% of the time he guesses "cat", determine the ratio of the number of times he guessed "dog" to the number of times he guessed "cat"

4. Suppose "a", "b", "c" are non zero real numbers. Define $h(x) = \frac{ax+b}{bx+c}$ for which $x \neq \frac{-c}{b}$. Determine all triples (a,b,c,) for which $h(h(x)) = x$ for every real number "x" with $x \neq \frac{-c}{b}$ and $h(x) \neq \frac{-c}{b}$

5. Suppose “ x ” and “ y ” are angles with $\tan x = \frac{1}{m}$ and $\tan y = \frac{a}{n}$ for some positive integers “ a ”, “ m ”, and “ n ”. Determine the number of positive integers $a \leq 50$ for which there are exactly 6 pairs of positive integers (m, n) with $X + Y = 45^\circ$

$$\left(\text{Note: } \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \right)$$